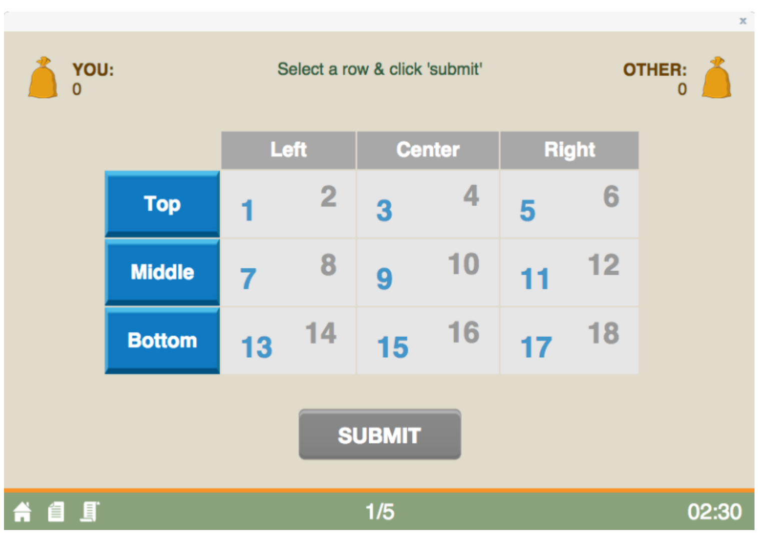
Matrix: Instructor Specified

Teaching Note | MobLab x Harvard Business Impact

Primary Topic:	Game Theory, Strategy, Industrial Organization. Configurable for any two-player normal-form game
Typical Duration:	Set-up: 5-10 min | Game play: 5-15 min | Debrief: 10-20 min
Group Size:	Pairs; scales to any class size
Difficulty Level:	Introductory through Advanced — depends entirely on the matrix you configure
Prerequisites:	Varies by matrix. Introductory game theory helpful. Students only need to understand that payoffs depend on both players' choices simultaneously.

1. Simulation Overview

The Matrix: Instructor Specified game is MobLab's most flexible teaching tool for game theory. You define the payoff matrix, from a 2x2 to a 5x5 grid, and students play the resulting two-player simultaneous-move game. One student is the Row player, the other is the Column player. Both choose an action simultaneously without seeing the other's choice; payoffs are determined by the cell at the intersection of their choices.



This note does not prescribe a single game or teaching objective. Instead, it serves as a reference for setting up the tool, a catalog of the most used matrix configurations and what they teach, and guidance on how to structure play and debrief regardless of which game you run. The right matrix for your class depends on what concept you want to illustrate — the sections below help you match your teaching goal to a configuration.

What makes this game different: Unlike MobLab's other pre-built games, the Matrix game has no fixed narrative or payoff structure. You are the designer. The game is as simple or complex as the matrix you specify. This makes it equally suited to a first-day introduction to strategic thinking and an advanced seminar on mixed-strategy equilibria or infinitely repeated games.

Learning Objectives

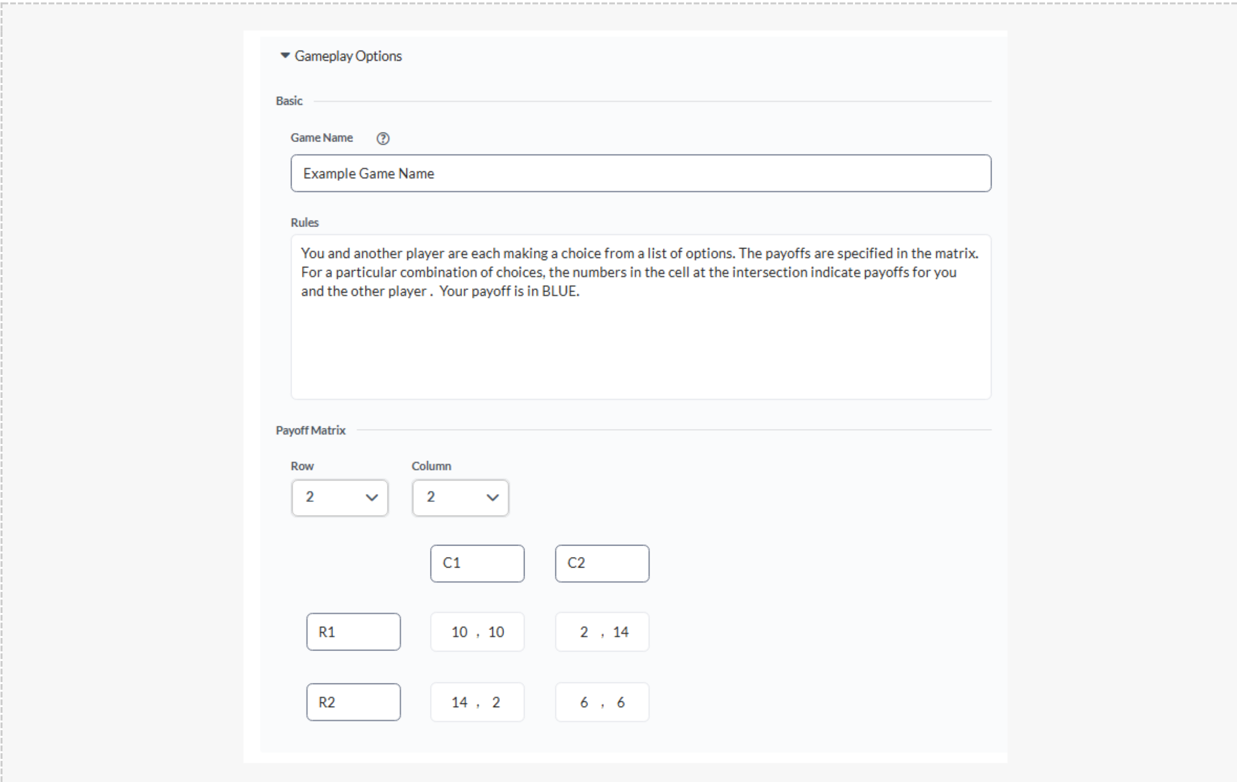
Depending on the matrix configuration, students will be able to:

- Nash Equilibrium: Identify Nash equilibria in pure strategies by finding cells where neither player can improve their payoff by unilaterally changing their action.
- Dominant Strategies: Identify strictly and weakly dominant strategies and apply iterated elimination of dominated strategies to solve games with a unique equilibrium.
- Mixed Strategy Equilibrium: In games with no pure-strategy Nash equilibrium, calculate the mixed-strategy equilibrium and observe whether actual play converges toward it over repeated rounds.
- Multiple Equilibria and Coordination: In coordination games (Stag Hunt, Battle of the Sexes), identify multiple pure-strategy Nash equilibria, distinguish between them by risk and payoff dominance, and analyze the coordination problem that arises when players must choose without communication.
- Repeated Interaction and Cooperation: In games where defection is the one-shot equilibrium (Prisoner's Dilemma), observe how indefinite repetition and communication can sustain cooperative outcomes — and connect this to the Folk Theorem.
- Flexibility for Custom Teaching Goals: Configure any payoff structure to support your specific learning objectives, whether that is illustrating a concept from a textbook, replicating a classic experiment, or designing a novel scenario relevant to your course.

2. Configuring Your Matrix

Matrix Dimensions and Action Labels

Navigate to the game configuration screen and set the number of Row actions and Column actions independently, they do not need to match. You can use any combination from 2 to 5 actions per player. For each action, enter a custom label (e.g., 'Cooperate / Defect', 'Rock / Paper / Scissors', 'Left / Center / Right'). You can also customize the text that appears on the student rules page to frame the game in your course context.



Entering Payoffs

Click each cell to enter the Row player's payoff and the Column player's payoff. Payoffs can be any integer, positive, negative, or zero. A few practical notes:

- Keep payoffs small and integers where possible.
- Symmetric payoffs (same structure for both players) are easier to explain and are appropriate for most introductory applications. Asymmetric payoffs are useful for modeling bargaining power, role differences, or heterogeneous preferences.
- Test the matrix in Preview Activity before class to verify that the payoffs display correctly and that the Nash equilibrium is where you expect it.

Instructor Tip: Write the payoff matrix on the board (or display it on screen) before students play, and leave it visible throughout. For introductory students, walk through one cell explicitly: 'If Row plays X and Column plays Y, Row earns A and Column earns B.' This ensures every student understands the payoff structure before making decisions.

Rounds and Repetition

Configuration	Recommended Setting
Minimum Rounds	Set to 1 for one-shot play. Set to 3-10 for repeated interaction with fixed pairs. For indefinitely repeated play, set Minimum Rounds to

Configuration	Recommended Setting
	a base number (e.g., 5) and use Ending Probability to simulate an infinite horizon.
Ending Probability (%)	100% = game ends after exactly Minimum Rounds. Less than 100% = each round has that probability of being the last, creating an indefinitely repeated game.
Role Assignment	Row and Column roles are fixed within a game. Use Replay > Rotate Role to let students experience both sides across games.
Replay Options	Random: reshuffle both roles and partners. Fixed: keep partners, randomize roles. Fixed Role: keep roles, randomize partners. Rotate Role: swap Row/Column assignments. Replay is the key tool for comparing one-shot vs. repeated outcomes.
Chat	Disabled by default. Enabling chat allows pre-play communication, which typically increases cooperation in coordination and social dilemma games. Running the same matrix with and without chat isolates the effect of communication.
Robot Players	Robots randomly play any available action. Useful for filling unpaired slots but will not best respond strategically.

3. Common Matrix Configurations

The table below summarizes the most commonly used matrix configurations, the equilibrium concept they illustrate, and the teaching context where each works best. Detailed notes on several configurations follow.

Game	Size	Key Equilibrium Concept	Best For
Prisoner's Dilemma	2x2	Dominant strategy / Nash equilibrium	Dominant strategies, social dilemma, cooperation
Stag Hunt	2x2	Two pure-strategy NE (risk vs. payoff dominant)	Coordination failure, equilibrium selection
Battle of the Sexes	2x2	Two pure-strategy NE + one mixed-strategy NE	Coordination with conflicting preferences
Matching Pennies	2x2	No pure-strategy NE / unique mixed-strategy NE	Mixed strategies, randomization
Rock, Paper, Scissors	3x3	Unique mixed-strategy NE (1/3, 1/3, 1/3)	Mixed strategies, cycling, larger action sets
Hawk-Dove	2x2	Two pure-strategy NE + one mixed-strategy NE	Conflict, resource competition, evolutionary games
Custom 4x4 / 5x5	4x4 or 5x5	Instructor-defined	Advanced courses, novel scenarios, iterated dominance

Prisoner's Dilemma

The canonical social dilemma. Each player has a dominant strategy (Defect) that leads to a Nash equilibrium worse for both players than mutual cooperation. The standard payoff structure: (C,C) > (D,C) for the defector, (C,C) > (D,D) for both, making unilateral cooperation exploitable.

- One-shot play: Defect/Defect dominates; use to introduce dominant strategies and Nash equilibrium.
- Repeated play (Ending Probability < 100%): Cooperation becomes sustainable; use to demonstrate the Folk Theorem and how indefinite repetition enables collusion.
- Add chat: communication typically increases cooperation even in a one-shot setting; use to discuss cheap talk and pre-play communication.

Stag Hunt

A coordination game with two pure-strategy Nash equilibria: (Stag, Stag) is payoff-dominant but risky; (Hare, Hare) is risk-dominant but inefficient. Unlike the Prisoner's Dilemma, both players want to coordinate — but uncertainty about the other player's choice may trap groups at the inferior equilibrium.

- Use to introduce the distinction between payoff dominance and risk dominance, and to demonstrate how strategic uncertainty — not individual self-interest — causes coordination failure.
- Run back to back with chat enabled: coordination on Stag typically increases dramatically, illustrating that cheap talk resolves coordination problems more easily than social dilemmas.
- Connects naturally to the Minimum Effort game — both involve the same underlying structure of coordination risk.

Battle of the Sexes (Bach or Stravinsky)

Two pure-strategy Nash equilibria (both attend the same event) and one mixed-strategy equilibrium. Players prefer to coordinate but have conflicting preferences about which equilibrium to reach. The mixed-strategy equilibrium produces frequent miscoordination.

- Use to introduce games with multiple equilibria where the equilibrium selection problem is not resolved by payoff ranking — both pure-strategy equilibria are Pareto optimal, but players disagree about which one to play.
- Repeat with rotating roles: students experience both the preferred and non-preferred equilibrium, generating discussion about bargaining, commitment, and social norms as equilibrium selection devices.
- The mixed-strategy equilibrium is particularly counterintuitive here: each player randomizes in a way that makes the other player indifferent, producing coordination failures more than half the time. Ask students: does this match what you observed?

Matching Pennies

A zero-sum game with no pure-strategy Nash equilibrium. The unique equilibrium requires both players to randomize 50/50. Any systematic pattern in one player's choices can be exploited by the other — making this the canonical illustration of the value of unpredictability.

- Use to introduce mixed-strategy Nash equilibrium in the cleanest possible setting. The equilibrium prediction is stark and easily verifiable: does each action appear approximately 50% of the time?
- With multiple rounds, students who fail to randomize adequately will be exploited by observant opponents, motivating the need for genuine randomization in adversarial settings.
- Connects to real-world applications: penalty kicks in soccer, play-calling in American football, inspection games in tax compliance and workplace monitoring.

Rock, Paper, Scissors (3x3)

Extends Matching Pennies to a three-action setting. The unique Nash equilibrium is for each player to play each action with equal probability (1/3). With three actions, students can explore iterated dominance (there are no dominated strategies here), asymmetric versions, and the visual patterns in action frequency results.

- Use to show that mixed-strategy equilibria generalize to larger action sets and that the equilibrium still requires uniform randomization when the game is symmetric.
- Modify the payoffs to create an asymmetric version (e.g., Rock beats Scissors for a higher payoff than the others) and observe whether equilibrium play shifts toward the higher-payoff action.

Custom 4x4 and 5x5 Matrices

For advanced courses, the larger matrix formats open up a wider range of concepts: games solvable by iterated elimination of strictly dominated strategies, games with dominant strategy equilibria buried in larger action sets, or scenarios where students must work through multiple rounds of elimination to find the solution. Design your matrix to require exactly the number of elimination rounds you want students to practice.

- Iterated elimination exercise: design a 4x4 matrix where the first column and last row are strictly dominated, reducing the game to a 3x3, then repeat until a unique equilibrium is identified. Students work through the logic step by step.
- Real-world framing: a 4x4 or 5x5 matrix can represent oligopoly pricing with multiple price points, political candidates choosing policy positions, or platform strategies — wherever there are more than two discrete options.

4. Pedagogical Instructions

Pre-Game Briefing

Before play, make sure students understand:

1. The structure: both players choose simultaneously without seeing the other's choice. There is no order of play advantage.

2. How to read the matrix: Row player's payoff is listed first in each cell; Column player's payoff is second. (Or display both payoffs clearly in your matrix design.)
3. The goal: maximize your own payoff, given uncertainty about what the other player will do.
4. Walk through one or two cells explicitly using your matrix before starting.

> **Instructor Tip:** Before revealing the Nash equilibrium prediction, ask students to write down their planned strategy and why. Collecting these (or asking for a show of hands) lets you compare pre-play intentions to actual behavior in the debrief, and generates discussion about whether students were thinking strategically or relying on intuition.

During Play

- For one-shot games: a single round is sufficient. Use Replay > Random to re-pair students and run additional rounds with different partners, treating each as an independent one-shot observation.
- For repeated games: run 5-10 rounds with fixed pairs. Watch for cooperation in social dilemma games increasing over rounds, or coordination improving in coordination games as players learn each other's tendencies.
- The Compare button (after replaying) lets you display two or more sessions side by side — powerful for showing how behavior changes from one-shot to repeated play, or from no-chat to chat.

> **Instructor Tip:** For repeated games, use an Ending Probability below 100% rather than a fixed number of rounds. When students know the exact final round, backward induction unravels cooperation in the last round and cascades backward. An indefinite horizon keeps the incentive to cooperate alive throughout play, which is usually more instructive.

Facilitator Checklist

5. Design your matrix and enter it in the configuration screen before class. Note the Nash equilibrium/equilibria for reference during the debrief.
6. Test with Preview Activity to verify payoffs display correctly and the game runs as intended.
7. Set round structure: one-shot (Minimum Rounds = 1, Ending Probability = 100%) or repeated (adjust as needed).
8. Decide whether to enable chat. Configure chat in the replayed game, not the baseline.
9. After play, navigate to Results > Graphs to show the action frequency and outcome frequency charts.
10. Use Replay and Compare to show results across treatment conditions side by side.

5. Debriefing & Discussion

The debrief questions will depend on which matrix you ran. The following questions are broadly applicable and can be adapted to any configuration:

Discussion Questions — Any Matrix

- What strategy did you choose, and why? Were you trying to maximize your own payoff, react to what you expected your partner to do, or something else?
- Look at the results. Did the outcome match the Nash equilibrium prediction? If not, what explains the gap between theory and behavior?
- In repeated-play sessions: did your strategy change across rounds? What triggered that change? Your own payoff, your partner's choices, or something else?
- If communication was enabled: how did knowing your partner's intentions change your strategy? Why does cheap talk (non-binding communication) influence behavior here?

Discussion Questions — Social Dilemma Games (Prisoner's Dilemma)

- Defecting is the dominant strategy. Did you defect? If you cooperated, why — and what does that tell us about the limits of the self-interest assumption?
- In the repeated version: did cooperation increase? What sustained it — fear of retaliation, genuine preference for the cooperative outcome, or something else?
- What real-world situations have the same structure? (Arms races, price competition, climate agreements, team effort.)

Discussion Questions — Coordination Games (Stag Hunt, Battle of the Sexes)

- There are two Nash equilibria. Which one did your group converge on? Was that the payoff-dominant equilibrium or the risk-dominant one?
- What made it difficult to coordinate on the better equilibrium without communication? What mechanisms in real organizations help people coordinate on efficient outcomes?
- With chat enabled: how did pre-play communication affect coordination? Why is it easier to communicate your way to the good equilibrium here than in the Prisoner's Dilemma?

Discussion Questions — Mixed Strategy Games (Matching Pennies, Rock Paper Scissors)

- Did you try to randomize? Did it feel natural or forced? How did you decide what to play each round?
- Look at your action frequencies over rounds. Were you close to the equilibrium mixture? If not, were you being exploited?
- Where in the real world do you need to be unpredictable? (Penalty kicks, play-calling, security inspections, bluffing in poker.)

6. Results & Data Interpretation

Reading the Results Dashboard

- Payoff Matrix Display (top of Graphs tab): Shows your configured matrix with the Nash equilibrium cell(s) highlighted. Use this to anchor the debrief — point to the equilibrium cell and ask whether that's where most outcomes landed.

- Action Frequency Chart: Shows the percentage of Row and Column players choosing each action per round. In dominant-strategy games, you expect convergence toward the dominant action. In mixed-strategy games, look for convergence toward the equilibrium mixture. In coordination games, look for which equilibrium attracted the most play.
- Outcome Frequency Chart: Shows the percentage of each action pair (e.g., Cooperate/Cooperate, Cooperate/Defect) per round. This is the most diagnostic chart for social dilemma games — it shows directly how often cooperation was achieved and whether it increased with repetition.
- Tables Tab: Displays the payoff matrix and frequency of each outcome per round, plus cumulative totals. Sort by outcome frequency to quickly identify which cell dominated play.
- Raw Data Tab: Individual player choices, groups, and round numbers. Useful for identifying specific pairs that cooperated throughout, or pairs that started cooperative and defected in the final rounds.

Using Compare Across Sessions

After replaying a game with different parameters, click the Compare button to display two sessions side by side. This is the most powerful debriefing tool for this game — use it to show:

- One-shot vs. repeated play: Does cooperation increase with repetition? Does the increase depend on the ending probability?
- No-chat vs. chat: Does pre-play communication shift action frequencies toward cooperation or the payoff-dominant equilibrium?
- Different matrices: How does behavior change when the payoff structure changes — for example, increasing the temptation payoff in the Prisoner's Dilemma, or making one equilibrium more payoff-dominant in the Stag Hunt?

7. Suggested Teaching Sequences

The Matrix game is most powerful when it anchors a sequence of related activities. A few proven sequences:

Game Theory Introduction (2-3 sessions)

- Session 1: Run the Matrix game as a Prisoner's Dilemma, which shows the formal payoff matrix.
- Session 2: Run repeated Prisoner's Dilemma (Ending Probability < 100%) and compare results to session 2 with the Compare tool.

Equilibrium Selection and Coordination

- Run Stag Hunt (no chat) and observe coordination failures. Then replay with chat enabled and compare. Discuss why communication works here.

Mixed Strategies

- Run Matching Pennies first (2 actions) to derive the 50/50 equilibrium on the board.
- Run Rock, Paper, Scissors (3 actions) to derive the 1/3, 1/3, 1/3 equilibrium and compare to actual play.
- Optionally run an asymmetric version of RPS to show how equilibrium mixtures change when payoffs are unequal.

8. References & Further Reading

The following resources pair well with this game across a range of configurations:

- Nash, J.F. (1950). Equilibrium Points in N-Person Games. Proceedings of the National Academy of Sciences, 36(1): 48-49. The foundational paper introducing Nash equilibrium — the solution concept underlying every matrix game.
- Harsanyi, J.C. & Selten, R. (1988). A General Theory of Equilibrium Selection in Games. MIT Press. The reference for payoff dominance and risk dominance in games with multiple equilibria (Stag Hunt, Battle of the Sexes).
- Axelrod, R. (1984). The Evolution of Cooperation. Basic Books. The classic study of cooperation in repeated Prisoner's Dilemma — essential context for repeated-play sessions.
- Camerer, C.F. (2003). Behavioral Game Theory: Experiments in Strategic Interaction. Princeton University Press. The definitive empirical treatment of how people actually play matrix games, including Prisoner's Dilemma, coordination games, and mixed-strategy games.
- Fudenberg, D. & Tirole, J. (1991). Game Theory. MIT Press. The graduate-level reference for formal derivations of Nash equilibrium, mixed strategies, and repeated games.