Stackelberg Competition

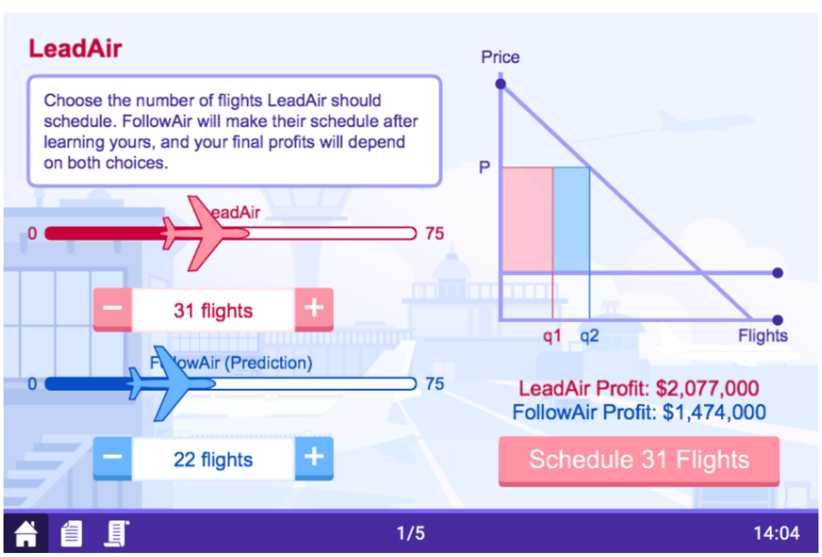
Teaching Note | MobLab x Harvard Business Impact

Primary Topic:	Industrial Organization, Game Theory, Microeconomics, Strategy
Typical Duration:	Set-up: 10 min | Game play: 10 min | Debrief: 10-15 min
Group Size:	Pairs (LeadAir and FollowAir); scales to any class size
Difficulty Level:	Intermediate / Advanced
Prerequisites:	Familiarity with Cournot competition and best-response functions. Works best after covering simultaneous quantity competition and before or alongside a lecture on sequential games and backward induction.

1. Simulation Overview

In this simulation, two airlines compete on a new route by choosing how many flights to schedule. The game is played sequentially: LeadAir moves first, choosing its flight quantity. FollowAir observes this choice before making its own decision. Market price is then determined by total output according to a linear demand curve: $P(Q) = 150 - qL - qF$, where qL is LeadAir's quantity and qF is FollowAir's. Both airlines face the same constant marginal cost of $30 per flight.

Both roles have access to interactive profit calculators. LeadAir can move both sliders to explore how profit changes with its own quantity at any assumed FollowAir response. FollowAir sees LeadAir's actual choice locked in and can only move its own slider — the best response function is immediately visible as the profit-maximizing point on FollowAir's side. This makes the logic of backward induction tangible: FollowAir's decision is a simple optimization problem with no strategic uncertainty.



The parameters are chosen to produce clean integer equilibria: the Stackelberg subgame-perfect Nash equilibrium is $q_L = 60$, $q_F = 30$ (market price \$60); the Cournot simultaneous-move equilibrium is $q_i = 40$ for both firms (market price \$70). This makes the first-mover advantage directly visible in the data — leaders who play optimally earn \$1,800K vs. \$1,600K in Cournot, while followers earn \$900K vs. \$1,600K.

Equilibrium Comparison — Default Parameters

The table below summarizes the three key outcomes for reference during the debrief:

Outcome	Leader Qty	Follower Qty	Price	Leader Profit	Follower Profit
Stackelberg (SPNE)	60	30	\$60	\$1,800K	\$900K
Cournot (Nash)	40	40	\$70	\$1,600K	\$1,600K
Collusive (Joint Max)	30	30	\$90	\$1,800K	\$1,800K

Key Observation: The Stackelberg leader earns more than in Cournot (\$1,800K vs. \$1,600K) by committing to a higher output that forces the follower to reduce. The follower earns substantially less (\$900K vs. \$1,600K). Total output is higher and price is lower than Cournot, benefiting consumers.

Learning Objectives

By the end of this simulation, students will be able to:

- Subgame-Perfect Nash Equilibrium (SPNE): Derive the Stackelberg equilibrium using backward induction, solving the follower's optimization problem first to obtain the best-response function, then substituting it into the leader's profit function to find the optimal leader quantity.
- First-Mover Advantage: Explain why sequential commitment to a higher output level increases the leader's profit relative to the simultaneous Cournot equilibrium, and why the ability to credibly commit is essential to this advantage.
- Best-Response Functions: Use the follower's best-response function to show how the leader effectively constrains the follower's output choice by its own output decision and identify the leader's optimal point along the follower's best-response curve.
- Comparing Market Structures: Compare equilibrium outcomes (quantities, price, profits) across Stackelberg, Cournot, and joint profit maximization, and explain the welfare implications of each.

2. Game Mechanics

Backward Induction — Solving the Game

The Stackelberg equilibrium is found by solving backward from the follower's decision:

Step 1 — Follower's Best Response

FollowAir maximizes its profit given LeadAir's choice qL. FollowAir's profit is: pi_F = (150 - qL - qF - 30) x qF = (120 - qL - qF) x qF. Taking the first-order condition and solving: qF* = (120 - qL) / 2 = 60 - qL/2. This is FollowAir's best-response function — for every LeadAir quantity, it tells us the profit-maximizing FollowAir response.

Step 2 — Leader's Optimization

LeadAir anticipates that FollowAir will best-respond. Substituting qF* into LeadAir's profit: pi_L = (150 - qL - (60 - qL/2) - 30) x qL = (60 - qL/2) x qL. Maximizing over qL gives the first-order condition: 60 - qL = 0, so qL* = 60. Substituting back: qF* = 60 - 60/2 = 30.

Step 3 — Equilibrium Payoffs

With qL = 60 and qF = 30, total output Q = 90, price P = 150 - 90 = 60. LeadAir profit = (60 - 30) x 60 = $1,800K. FollowAir profit = (60 - 30) x 30 = $900K.

> **Instructor Tip:** Walk through this derivation on the board before the game. Students who understand the math before playing are far better at recognizing whether they are playing optimally during the game, which produces richer debrief conversations about why some leaders chose less than 60 and whether followers actually best-responded.

The Profit Calculator

Both roles use an on-screen profit calculator with two sliders and a live demand curve chart:

- LeadAir slider (top): Sets LeadAir's quantity. The displayed profit assumes the FollowAir quantity shown on the bottom slider. Moving the top slider shifts LeadAir's position along the demand curve.
- FollowAir prediction slider (bottom): Sets a hypothetical FollowAir response. LeadAir can use this to anticipate how FollowAir will respond to any given leader quantity. The profit chart updates live.
- For FollowAir: the top slider is locked to LeadAir's actual choice. Only the bottom slider is free. FollowAir sees directly how its profit changes with its own quantity given LeadAir's committed output — making the best response essentially a visual optimization.

[INSERT SCREENSHOT HERE]

Alt Text: "MobLab Stackelberg LeadAir screen showing two sliders (LeadAir quantity on top, FollowAir prediction on bottom), a live demand curve with profit areas shaded for each firm, and the Schedule Flights button."

3. Classroom Setup

Configuration	Recommended Setting
Number of Rounds	Set to 1 for the first game. Use Replay (not Copy) for subsequent rounds — this links results for multi-round comparison. Replay > Random reshuffles both roles and pairings; Replay > Fixed Role reshuffles pairings only. Repetition without fixed pairings produces faster convergence to SPNE.
Round Duration	Default: No time limit. Students need time to explore the profit calculator. For experienced classes or time constraints, set to 180 seconds.
Market Parameters	Default: P(Q) = 150 - qL - qF, MC = 30. These produce clean integer equilibria (Stackelberg: 60/30, Cournot: 40/40). Do not change without updating the pre-game derivation.
Robot Players	Leader robot: samples from a binomial distribution centered on the Stackelberg equilibrium quantity (60). Follower robot: always best-responds exactly. A follower robot paired with a human leader is particularly useful for isolating whether leaders play optimally.
Technology Required	Laptop with an up-to-date web browser. The profit calculator is graphically intensive — tablets may be slower.

4. Pedagogical Instructions

Pre-Game Briefing

Spend 5-10 minutes before play covering:

- Explain the sequential structure: LeadAir moves first and its choice is irrevocable before FollowAir decides. This commitment is the source of the first-mover advantage — it is not just about moving first, but about the fact that FollowAir observes and reacts to LeadAir's committed quantity.
- Walk through the demand equation on the board: P = 150 - qL - qF, MC = 30. Ask students: if LeadAir chooses 60 and FollowAir best-responds, what quantity does FollowAir choose? Work through qF* = 60 - 60/2 = 30 together.
- Ask: should LeadAir choose 40 (the Cournot quantity) or 60 (the Stackelberg quantity)? Compare the profits in each case. With qL = 40 and qF best-responding at 40, LeadAir earns $1,600K. With qL = 60 and qF best-responding at 30, LeadAir earns $1,800K. The lesson: by committing to more output, LeadAir forces FollowAir to pull back.
- Explain the profit calculator interface before the game starts so students know how to use the sliders to explore scenarios.

> **Instructor Tip:** Before starting, ask students playing as LeadAir to write down their planned quantity and their reasoning. After the game, compare what they planned vs. what they chose. Many leaders initially plan to choose 40 (the Cournot quantity) before realizing they can do better — this trajectory is worth discussing explicitly.

The Gameplay Experience

Watch for the following dynamics across rounds:

- Early rounds — Cournot-like Leader Quantities: In the first game, many leaders choose quantities near 40 — the symmetric Cournot quantity — out of intuition or inertia from studying simultaneous games. Followers typically best-respond well (the calculator makes this easy), so actual outcomes often fall near or above the Cournot equilibrium but below Stackelberg.
- Learning across rounds — Leader quantities increase: As leaders experience that choosing 40 leaves profit on the table (the Replay results show their earnings below the Stackelberg benchmark), they increase their output in subsequent rounds. Followers, observing higher leader quantities, respond by reducing their own.
- Convergence pattern: By round 3-5, most leader quantities cluster in the 50-70 range, and most follower quantities are close to their best-response values. Full convergence to exactly qL = 60, qF = 30 is less common but the directional trend is usually clear.
- Role asymmetry: The follower role is easier — the profit calculator essentially solves the problem visually. Watch for leaders who choose too low (leaving money on the table) and leaders who choose too high (overshooting and reducing their own profit as price collapses). Both are useful discussion cases.

Facilitator Checklist

1. Set up the game in a playlist with Number of Rounds = 1 before class. Prepare Replay configurations (Random and/or Fixed Role) for subsequent rounds.
2. Direct students to video instructions before class via the Instructions link.
3. Derive the Stackelberg equilibrium on the board before starting (10 min).
4. Open the game and wait for all students to join before clicking Begin.
5. After Game 1 concludes, use Replay (not Copy) to run subsequent rounds for linked results.
6. Before the debrief, open the last game's results and navigate to Multi-Round Summary to show the convergence graphs.

5. Debriefing & Discussion

Plan for 10-15 minutes. Begin with written reflection, then show results. The most effective debrief sequence: Multi-Round Summary (convergence trend) then the Best Response Chart (which groups found the equilibrium) then the summary table (specific group outcomes vs. benchmarks).

Suggested Reflection Questions (before viewing results)

7. As LeadAir, how did you arrive at your quantity choice? Did you think about how FollowAir would respond?
8. As FollowAir, was your decision easy or difficult? What information did you rely on?
9. Can you think of a real industry where the order in which firms move matters?

Discussion Questions (after viewing results)

- Look at the Profits per Round graph. Did leader profits increase across rounds? What does that tell us about whether leaders learned to exploit the first-mover advantage?

- Look at the Best Response Chart. Most follower choices fall on or near the follower best-response function. Why is it easier for followers to best-respond than for leaders to play optimally?
- The leader earns more than in Cournot ($1,800K vs. $1,600K) but produces more output. Why does producing more output help the leader here? What is the leader actually committing to, and why does that commitment have value?
- The follower earns much less than in Cournot ($900K vs. $1,600K). Why is the follower made worse off by the sequential structure? Would FollowAir prefer to move simultaneously?
- Why is the irrevocability of the leader's choice essential? If LeadAir could change its output after FollowAir decided, would the first-mover advantage still exist?
- Think of real industries where firms move sequentially — capacity investment, retail location, platform entry. What features of those industries make the Stackelberg model more or less applicable?

Theoretical Connections

- Subgame-Perfect Nash Equilibrium and Backward Induction: The Stackelberg equilibrium is found by backward induction — solving the follower's problem first, then substituting the best-response function into the leader's problem. This is the canonical application of SPNE in extensive-form games, and the game makes the two-stage structure tangible.
- First-Mover Advantage in Quantity Competition: The leader's advantage comes from commitment — by irreversibly choosing a higher output, the leader credibly forces the follower to reduce production. Without credibility (i.e., if the leader could revise its output), this advantage disappears. The game illustrates why commitment devices have strategic value.
- Cournot vs. Stackelberg: The gap between the two equilibria is the value of the first-mover advantage. The leader gains $200K relative to Cournot by moving first; the follower loses $700K. Total output rises from 80 (Cournot) to 90 (Stackelberg), benefiting consumers but hurting the follower. This welfare analysis connects to debates about merger policy, capacity preemption, and entry deterrence.
- Best-Response Functions: The Best Response Chart in the results screen plots both firms' best-response functions. The Stackelberg equilibrium is at the intersection of the leader's profit-maximizing point with the follower's best-response function — visually, it is where a leader-iso-profit curve is tangent to the follower's best-response line. This geometric interpretation connects the game to the theoretical treatment in industrial organization textbooks.
- Entry Deterrence: The logic of the Stackelberg model extends naturally to entry deterrence. An incumbent can deter entry by pre-committing to high capacity or output, making entry unprofitable for a potential entrant who best-responds to the incumbent's committed level. This connects the game to Spence (1977) and the broader literature on strategic entry barriers.

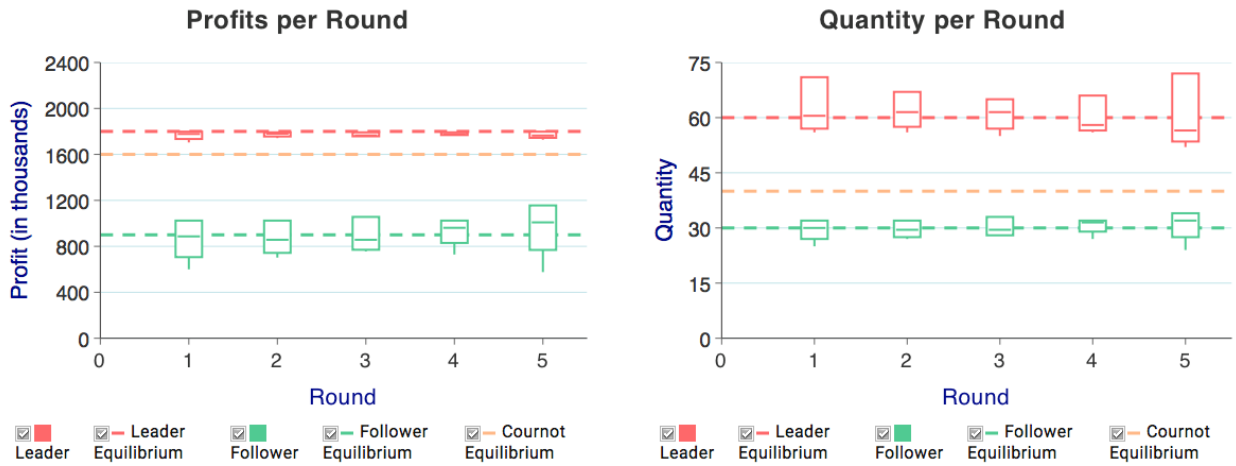
6. Results & Data Interpretation

Reading the Results Dashboard

Three views are available via the Go To dropdown:

Multi-Round Summary (Figure 1)

- Profits per Round (left panel): Box-and-whiskers plots of leader and follower profits for each round, with dashed reference lines at the Stackelberg and Cournot equilibrium profit levels. Leader profits should trend upward toward $1,800K; follower profits should trend downward toward $900K. Narrow boxes in later rounds indicate convergence.
- Quantity per Round (right panel): Same box-and-whisker format for quantities. Leader quantities should trend toward 60; follower quantities toward 30. Use the Cournot reference line (40/40) to show the directional shift.



> **Note:** If you used Replay to run multiple rounds, open the results from the last game played. Previous games in the series appear in the View dropdown at the top-left. Copying a game rather than replaying it will not link results for comparison.

Per-Round Summary Table and Best Response Chart

- Summary Table: Lists each group's leader quantity, follower quantity, market price, leader profit, and follower profit for the selected round. The top two rows show the Stackelberg and Cournot equilibrium benchmarks for direct comparison. Use this to identify which groups are close to SPNE and which are stuck at Cournot-like outcomes.
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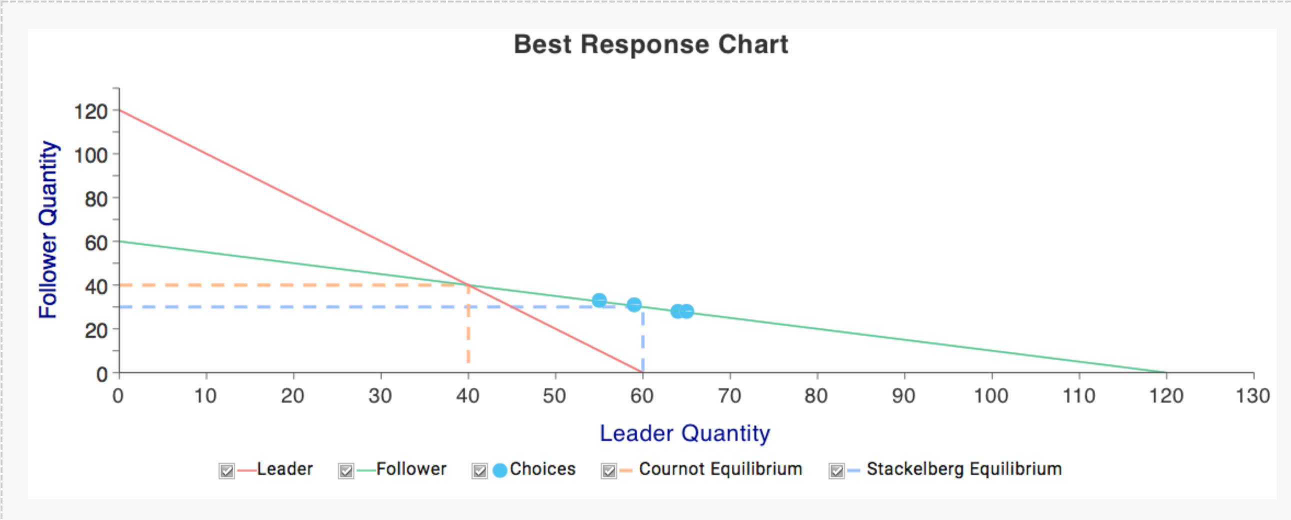
Group	Leader Quantity	Follower Quantity	Price	Leader Profit	Follower Profit
Equilibrium	60	30	60	1800	900
Cournot	40	40	70	1600	1600
1	64	28	58	1792	784
2	59	31	60	1770	930
3	55	33	62	1760	1056
4	65	28	57	1755	756

- Best Response Chart: Plots each group's (leader quantity, follower quantity) pair as a dot, with the follower's best-response function overlaid as a line. The Cournot and Stackelberg equilibria are marked. Most follower dots should fall on or near the best-response line — this is the result to highlight. The scatter of leader dots shows how far leaders are from the Stackelberg optimal of 60, and whether they are systematically undershooting or overshooting.



Common Outcome Patterns

- Leaders Undershoot in Round 1 (common): Most leaders choose quantities near 40-50, close to the Cournot quantity. This reflects the common failure to fully internalize the commitment logic. Follower profits are higher than Stackelberg predicts as a result.
- Followers Best-Respond Well (consistent): Because the profit calculator presents FollowAir's optimization as a transparent visual problem, most followers play close to their best-response function from round 1. The dot cloud in the Best Response Chart typically clusters tightly along the follower best-response line.
- Leader Quantities Increase Across Rounds: With Replay, leader quantities typically trend upward toward 60 as leaders learn to exploit the commitment advantage. This convergence is the most compelling debrief visual — the Profits per Round graph shows leader earnings trending toward the Stackelberg benchmark round by round.
- Occasional Overshooting: Some leaders choose quantities above 60, driving price so low that their profits fall below even the Stackelberg level. These cases are useful for discussing that the first-mover advantage has a limit — commitment to too much output reduces profit just as committing to too little does.

7. Extensions & Variations

Pairing with Cournot

The most powerful extension is running the Cournot game (simultaneous quantity choice) in the same class session and comparing results directly. The difference in leader profits between the two games is a clean, student-generated estimate of the first-mover advantage. Running Cournot first and Stackelberg second also gives leaders a concrete quantity anchor ($40, the Cournot equilibrium) to compare against the Stackelberg optimum.

Robot Follower Treatment

Pairing human leaders with a robot follower (which always best-responds exactly) isolates the leader's optimization problem. Any deviation from $q_L = 60$ is purely a leader decision error — followers are not the limiting factor. This treatment is useful for showing how much of the gap between actual and equilibrium outcomes is attributable to leader strategy vs. follower strategy.

Cross-Disciplinary Connections

- Industrial Organization — Capacity Preemption: Firms that can irreversibly invest in production capacity before competitors move are in a Stackelberg-like situation. The game motivates why incumbents sometimes invest in excess capacity — not to use it, but to credibly deter entry by making the market less profitable for a late entrant who best-responds to the incumbent's committed capacity.
- Business Strategy — First-Mover Advantage: The game provides the formal game-theoretic foundation for first-mover advantage in quantity competition. In strategy courses, connect this to empirical examples: Amazon's early commitment to scale in e-commerce, Walmart's commitment to low-price positioning, or Boeing's historical capacity pre-commitment in commercial aviation.
- Antitrust — Market Power and Sequential Competition: Sequential competition produces less symmetric outcomes than simultaneous competition. The follower's profit disadvantage raises questions about whether sequential market structures harm competition — and whether incumbent firms that can commit early use this advantage to maintain market power against potential entrants.

8. References & Further Reading

The following resources pair well with this experiment:

- Stackelberg, H. von (1934). Marktform und Gleichgewicht. Vienna: Springer. The original formulation of sequential quantity competition.
- Tirole, J. (1988). The Theory of Industrial Organization. MIT Press. Chapters on quantity competition, entry deterrence, and strategic commitment provide the formal treatment of concepts explored in this game.
- Spence, A.M. (1977). Entry, Capacity, Investment and Oligopolistic Pricing. Bell Journal of Economics, 8(2): 534-544. The foundational paper on capacity preemption as an entry deterrence strategy.
- Dixit, A. (1980). The Role of Investment in Entry Deterrence. Economic Journal, 90(357): 95-106. Connects Stackelberg commitment to strategic entry deterrence.
- Mankiw, N.G. (current edition). Principles of Economics. Cengage. Chapter on Oligopoly covers Cournot and the intuition for sequential competition accessible to introductory students.
- Varian, H.R. (current edition). Intermediate Microeconomics. Norton. Chapter on Game Theory and Oligopoly provides the formal derivation of both Cournot and Stackelberg equilibria used in this game.